Power Analysis Attacks on Falcon

The Hidden Parallelepiped Is Back Again: Power Analysis Attacks on Falcon [GMRR22]

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Overview

- Preliminaries
- Falcon signature scheme
- Power analysis on the preimage computation
- Hidden Parallelepiped attack on the trapdoor sampler
- Summary

Intro

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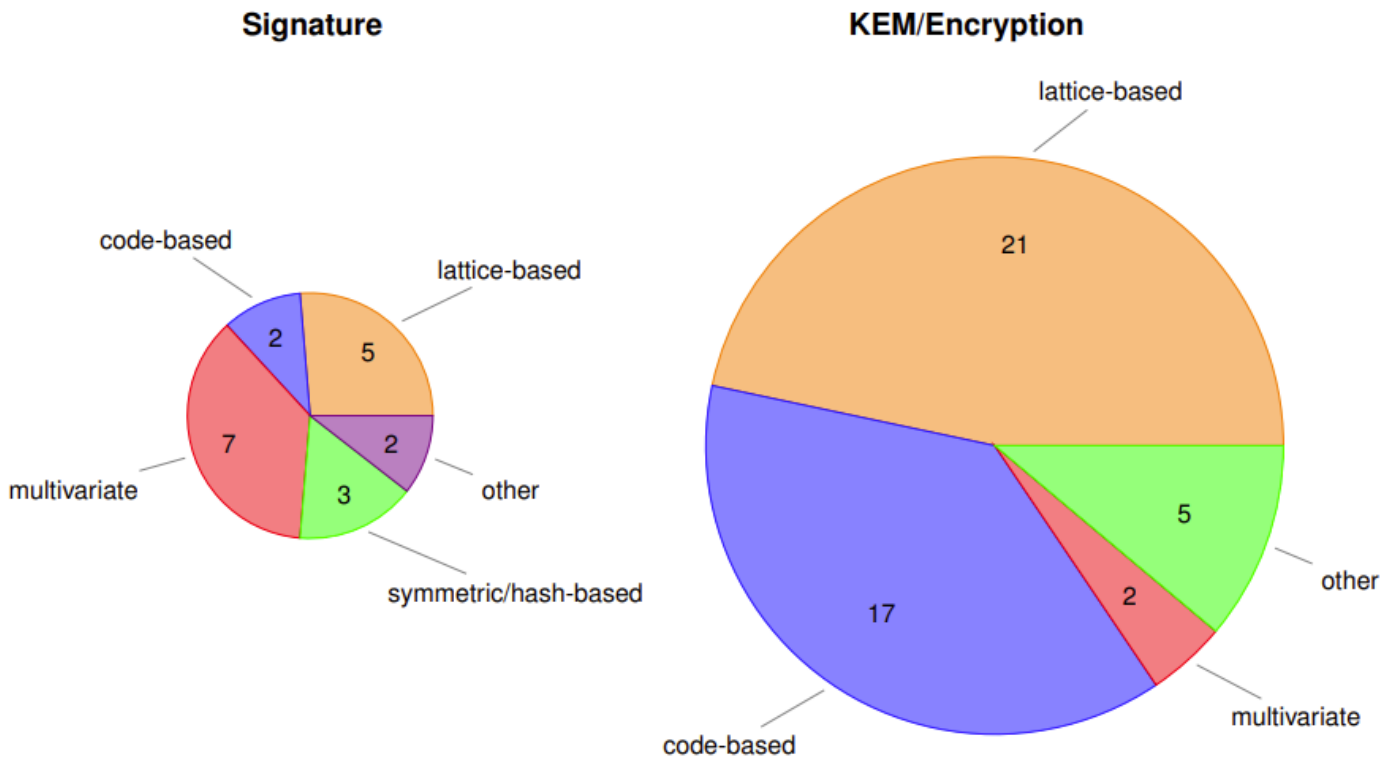
- It is a **hash-and-sign** lattice-based signature scheme.
- Falcon = GPV Framework + NTRU lattices + Fast Fourier sampling.
- Main advantage: **compactness**. Not known any post-quantum signature schemes getting $|pk| + |sig|$ = (bitsize of the public key + bitsize of a signature) to be as small as Falcon does.

NIST PQC Round #1 Submission



source: https://csrc.nist.gov/CSRC/media/Projects/Post-Quantum-Cryptography/documents/PQCrypto-April2018_Moody.pdf

Figure: Slide 808 Cryptography and Security course

NIST PQC 2022 Selected Algorithms

	Signature		KEM/Encryption	
Selected	CRYSTALS-DILITHIUM FALCON SPHINCS+	lattice lattice hash	CRYSTALS-KYBER	lattice

Next:

- KYBER becomes ML-KEM in FIPS 203 (Module-Lattice)
- DILITHIUM becomes ML-DSA in FIPS 204 (Module-Lattice)
- SPINCS+ becomes SLH-DSA in FIPS 205 (Stateless Hash)
- FALCON becomes FN-DSE (later)

Figure: Slide 809 Cryptography and Security course

Power Analysis

- **Threat model:** adversary has physical access to the device and captures EM and power measurements while the key-dependent computations are carried out.

"Power"
measure some physical quantity influenced by execution, e.g., power, EM, ...

- **Simple Power Analysis**
- **Differential Power Analysis**
- **Correlation Power Analysis**
- **Other leakage types**

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- Simple power analysis (SPA): The power usage tells what kind of operation is performed (e.g. square and multiply algorithm in decryption in RSA)
- **Differential Power Analysis (DPA):** a SCA that extracts secret information (e.g. signing key) by measuring a device's power consumption during computations.

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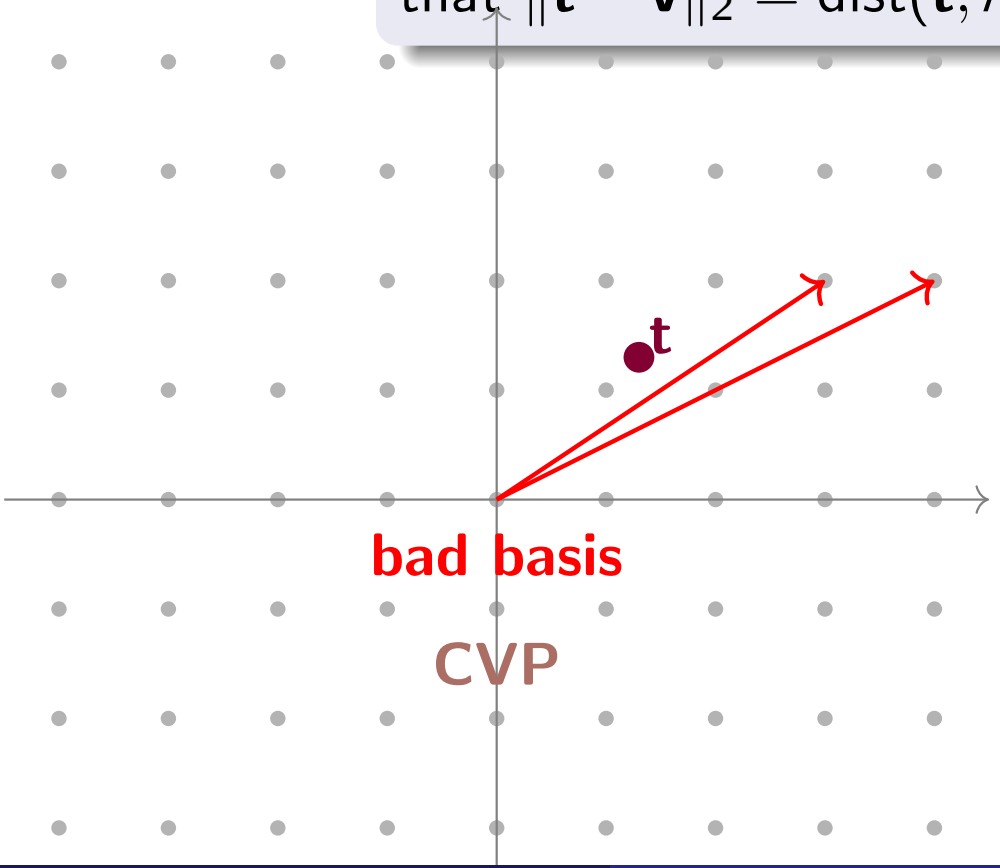
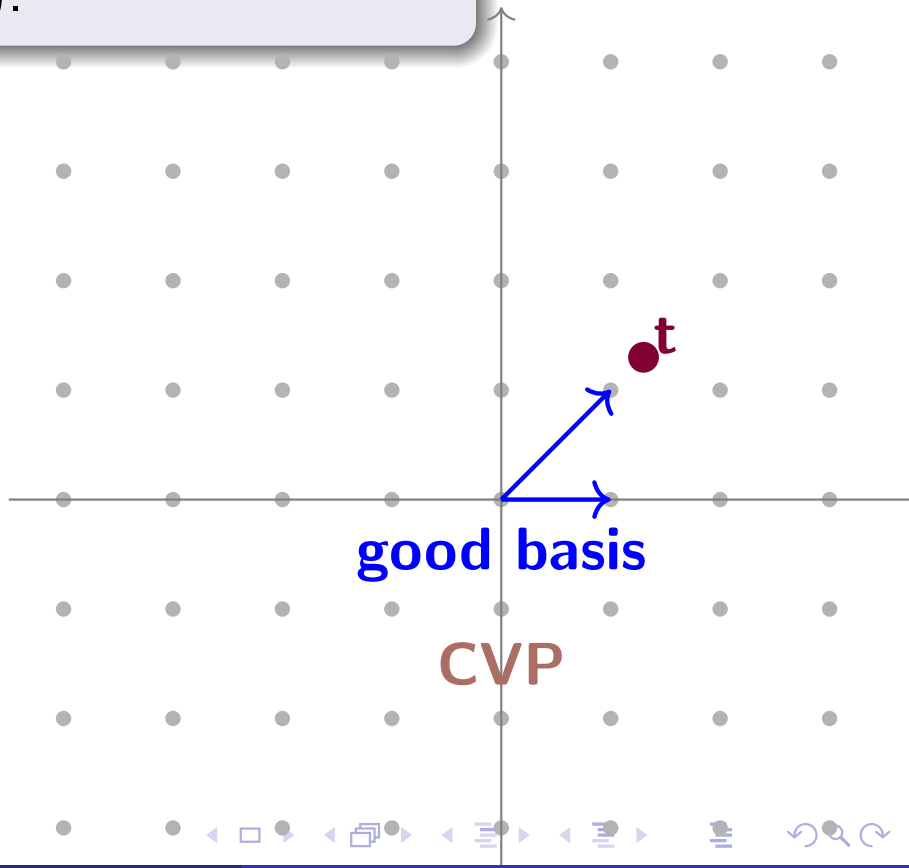
    - Decide which subkey guess correlates best to the measured traces.
  - Put together the best subkey guesses to obtain the full secret key.

Lattice-based cryptography [1/2]

Security from hard problem: Closest Vector Problem (CVP)

Given $\mathbf{B}$ a basis of a lattice $\Lambda \subset \mathbb{R}^n$ and a target vector $\mathbf{t} \in \mathbb{R}^n$, find a $\mathbf{v} \in \Lambda$ such that $\|\mathbf{t} - \mathbf{v}\|_2 = \text{dist}(\mathbf{t}, \Lambda)$.





Lattice-based cryptography [2/2]

- CVP is easy to solve with a good basis(:=short and reasonably orthogonal) but hard with a bad basis.
- **Signature scheme:**
  - Convert the message to sign to a vector $c \in \mathcal{R}^n$
  - Use the good basis (secret key) to solve CVP
  - Anyone can verify the signature $\mathbf{v}$ with a bad basis (public key)

Remark: It is hard to derivate the good basis from the bad basis.

Hidden Parallelepiped Problem (HPP)

- Transcript Analysis: each document, signature pair (d, s) reveals some information on the sk, at least it reveals that $Sign(sk, d) = (d, s)$. $\rightsquigarrow$ sufficiently many transcripts may reveal information about the signing key or how to forge another document.

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HPP

Recover $\mathbf{B}$ from independent samples drawn uniformly in $\mathcal{P}(\mathbf{B})$.

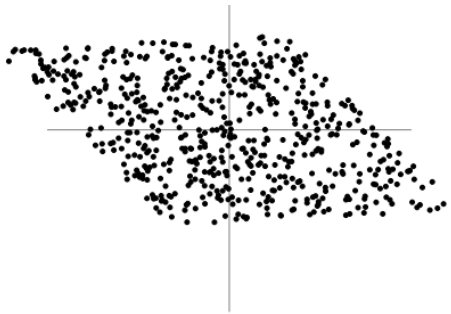

Fig. 1. The Hidden Parallelepiped Problem in dimension two.

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- **KeyGEN Falcon:** Draw $f, g \in \mathcal{R}$ with small coefficients, compute $F, G \in \mathcal{R}$ satisfying NTRU equation: $fG - gF = q \mod (x^n + 1)$. $h = gf^{-1} \mod q$ is the **public key** and f, g, F, G are the **secret keys**. Private Basis $B \in \mathcal{R}^{2n\times 2n}$, Public Basis $A \in \mathcal{R}^{2n}$ seen as $A := (1 \quad h^*) \in \mathcal{R}^2$ with $h^* := h(x^{-1})$.

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Falcon SIGN

Algorithm 1: FALCON.Sign(m, sk)

Input : A message m, a secret key $sk = (\hat{\mathbf{B}}, T)$
Output : A signature sig

1 $r \xleftarrow{\$} \{0,1\}^{320}$ uniformly
2 $c \leftarrow \texttt{HashToPoint}(r||m, q, n)$ $\quad c \in \mathcal{R}$
3 $\hat{\mathbf{t}} \leftarrow (\hat{c}, 0) \cdot \hat{\mathbf{B}}^{-1}$ • pre-image computation
4 **do**
5 $\quad \hat{\mathbf{v}} \leftarrow \texttt{ffSampling}(\hat{\mathbf{t}}, T)$ • trapdoor sampler
6 $\quad \hat{\mathbf{s}} \leftarrow (\hat{\mathbf{t}} - \hat{\mathbf{v}}) \cdot \hat{\mathbf{B}}$
7 **while** $\|\mathbf{s}\|^2 > \lfloor 2.42 \cdot n \cdot \sigma^2 \rfloor$ $\quad \sigma := \frac{1.17}{\pi\sqrt{2}} \cdot \sqrt{q \cdot \log\left(4n(1 + 2^{32} \cdot \sqrt{n/4})\right)}$;
8 **return** $sig := (r, \mathbf{s})$

- if $\|s\|^2 \leq \lfloor 2.42 \cdot n \cdot \sigma^2 \rfloor$, then the signature is accepted as valid. Otherwise, it is rejected.

Correlation Power Analysis on the preimage attack

- Original attack [KA21]: CPA on a polynomial multiplication in FFT between a public digest c and a private polynomial f: $\hat{c} \cdot \hat{f}$ since computing $\hat{t} \leftarrow (\hat{c}, 0) \cdot \hat{B}^{-1} \rightsquigarrow$ computing $\hat{c} \cdot \hat{f}$.

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  - **Partial knowledge on $\hat{f} := FFT(f)$ leads to KR**: by experiments, only the 8 MSB (obtained by CPA) of the mantissa are required to derive the key.
  - **halve the number of required traces** thanks to redundancy in multiplication of complex numbers.

Partial knowledge on $\hat{f} \implies$ KR

- **KR reduces** to recover f (actually $\hat{f}$ thanks to FFT^{-1}) from $\hat{c} \cdot \hat{f}$ because $h = g \cdot f^{-1} \mod q \rightsquigarrow$ recover g. As $fG - gF = q \mod (x^n + 1)$, $\rightsquigarrow$ recover F, G.

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- Float numbers in double precision (:=encoded over 64 bits):

$$(-1)^s \cdot 2^{e-1023} \cdot m$$

with [s=sign(1bit)| e= exponent(11bits)|m= mantissa(52bits)].

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- [KA21] attack's retrieve the whole mantissa VS [GMRR22] improvement: enough to recover only the 8MSB of the mantissa.

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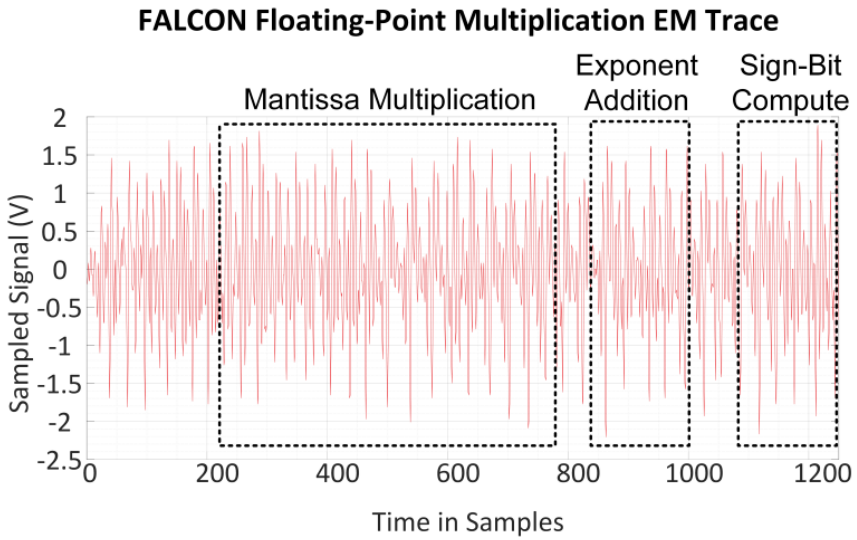

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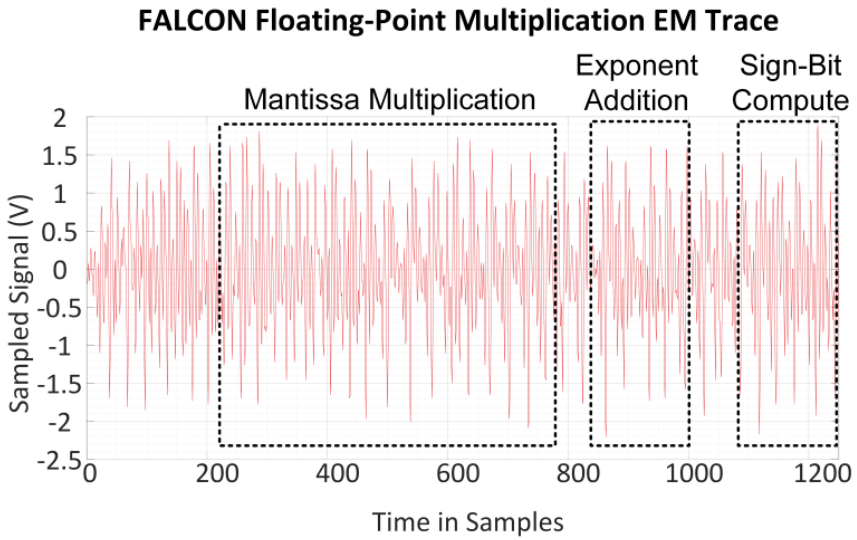

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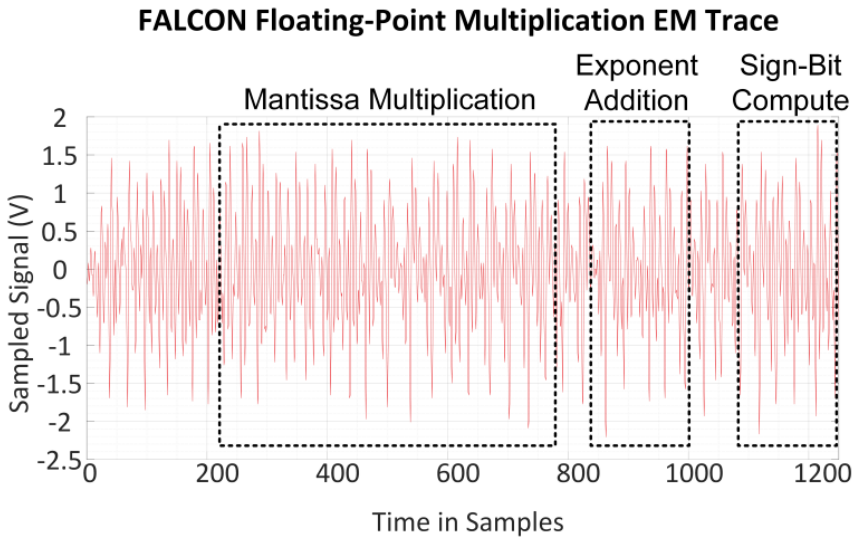

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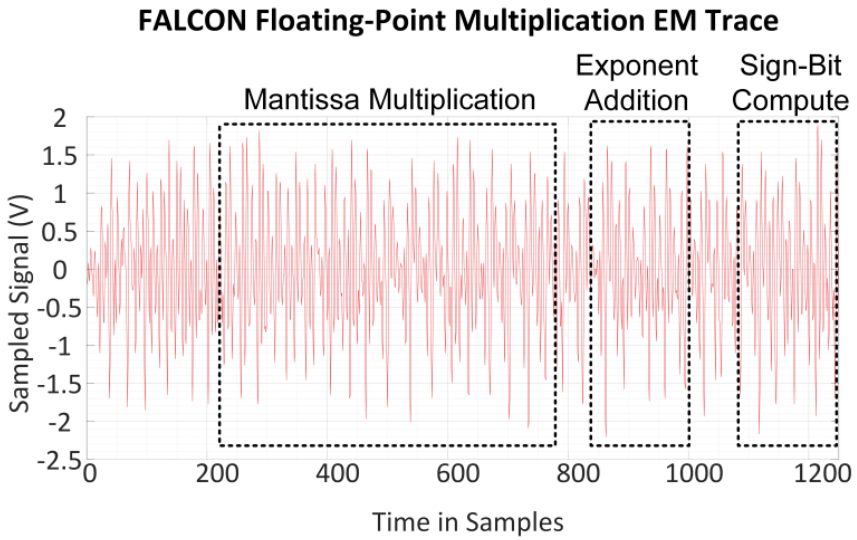

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- To recover its mantissa, enough to recover only its 8 MSBs.
- guess a value for $\mathrm{Re}(\hat{f}[i])$ (since $\hat{c}$ is known) for which it best correlate its Hamming weight with the Hamming weight of our record.

Halve the number of required traces

- We consider the j-th digest c_j (in floating point). Let $\hat{c}_j[i] = \mathrm{Re}(\hat{c}_j[i]) + i\ \mathrm{Im}(\hat{c}_j[i])$, and $\hat{f}[i] = \mathrm{Re}(\hat{f}[i]) + i\ \mathrm{Im}(\hat{f}[i])$.

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$$\begin{cases}\mathrm{Re}(\hat{c}_j[i]) \cdot \mathrm{Re}(\hat{f}[i]), & \mathrm{Im}(\hat{c}_j[i]) \cdot \mathrm{Re}(\hat{f}[i]) \\ \mathrm{Im}(\hat{c}_j[i]) \cdot \mathrm{Im}(\hat{f}[i]), & \mathrm{Re}(\hat{c}_j[i]) \cdot \mathrm{Im}(\hat{f}[i])\end{cases} \Rightarrow \text{Recover } \mathrm{Re}(\hat{f}[i]), \mathrm{Im}(\hat{f}[i])$$

- better than [KA21] where all 4 four multiplications were used to recover $\mathrm{Re}(\hat{f}[i])$.

CPA on the Preimage computation

	Number of Traces	$\mathbb{P}$(**Recover one intermediate value**)
KA21	2000	0.5
GMRR22	2000	0.9

Comparison of probability of recovering one intermediate value between [KA21] and [GMRR22] attack with all multiplication patterns with 2000 traces with data (10×16 coefficients).

Unravelling the Hidden Parallelepiped Problem with side-channel information

- New side-channel attack on Falcon's implementation
- Target the Gaussian sampler
- Recover the private key

Attack's procedure

1. SPA on `BaseSampler` to obtain samples of z^+
2. `ffSampling` & Deformed HPP to recover private key
3. Direct Key recovery & Lattice Reduction
4. Proposed countermeasure

Discrete Gaussian distribution

- Gaussian function centered at $c \in \mathbb{R}^n$ of standard deviation $\sigma \in \mathbb{R}_{>0}$, for any $x \in \mathbb{R}^n$,

$$\rho_{\sigma,c}(x) := \exp\left(-\frac{\|x-c\|^2}{2\sigma^2}\right)$$

- Discrete Gaussian function for any $z \in \Lambda$, $D_{\Lambda,\sigma,c(z)} := \frac{\rho_{\sigma,c}(z)}{\sum_{x\in\Lambda}\rho_{\sigma,c}(x)}$

SPA on BaseSampler - Recall

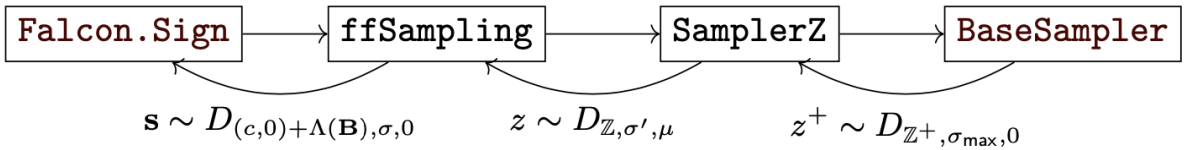

Figure: Signature's flowchart

Algorithm 2: `BaseSampler` ()

Output : An integer $z^+ \sim D_{\mathbb{Z}^+,\sigma_{\max},0}$

1 $u \leftarrow$ `UniformBits`(72)
2 $z^+ \leftarrow 0$
3 **for** $i \leftarrow 0$ **to** 17 **do**
4 $\quad z^+ \leftarrow z^+ + [\![u < \mathrm{RCDT}[i]]\!]$
5 **end**
6 **return** z^+

- $[[u < RCDT[i]]] := 1_{\{u<RCDT[i]\}}$
- RCDT= table of 18 numbers of 72 bits. $RCDT[i] := 2^{72} - CDT[i]$

SPA on BaseSampler - Weakness

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- Comparison $u < \mathrm{RCDT}[i]$ is a three 24-bits integer subtraction
- Result stored in 32-bits register
- What happens upon *underflows* on the last one?

- Register $= 1 \cdots 1 \,\|\, result$ (8 MSBs set to 1)
- Hamming weight increased by 8
- Retrieve the value of z^+ (especially $z^+ = 0$ or not)

Note: After `BaseSampler`, a sign is generated and $z \in \{0, 1\}$

SPA on BaseSampler - Trace

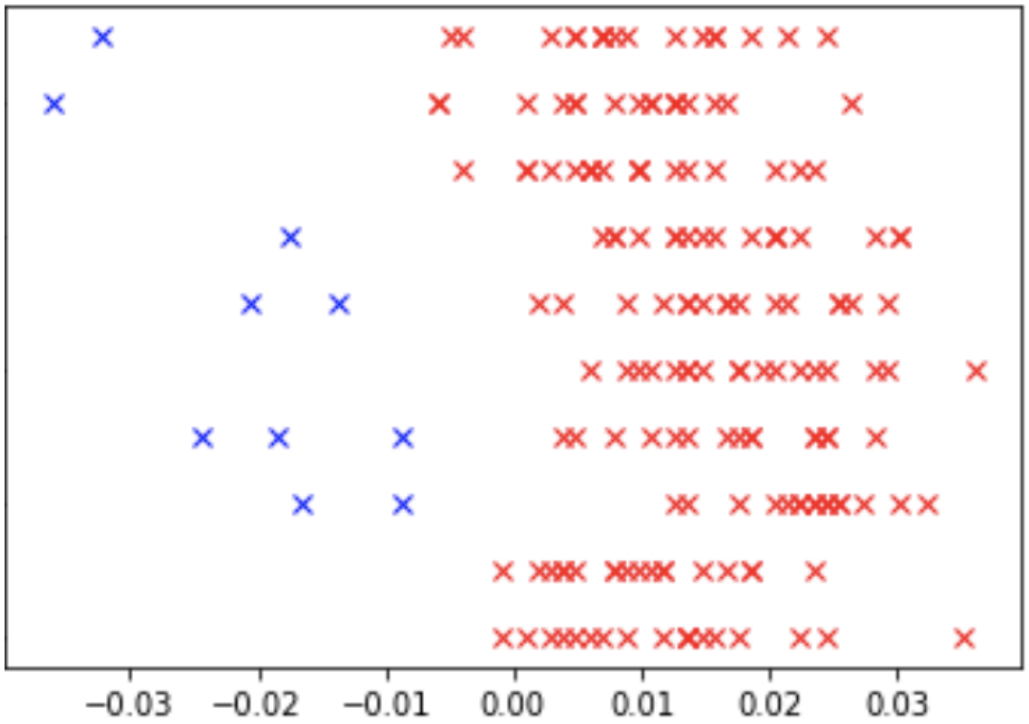


Figure: Power variation during the execution of the substraction. Each line is a different execution. Blue ticks stand for incrementation of z+, red ones for absence of it.

SPA on `BaseSampler` - Results

	Percentages of traces accurately classified
ELMO	100%
ChipWhisperer	94.2%
More efficient setup from [KH18]	100%

Table: Result of SPA attack on ELMO simulator, ChipWhisperer CPU and a more efficient setup from Kim and Hong in [KH18]

ffSampling

Input: target vector $\mathbf{t_n}$ and a private basis $\mathbf{B}$

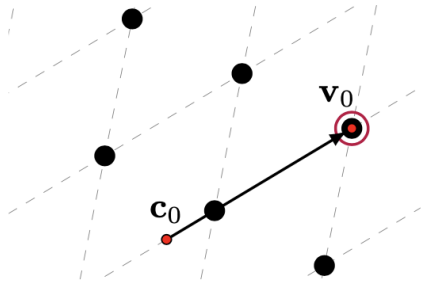
Output: $\mathbf{v} \sim D_{(c,0)+\Lambda(\mathbf{B}),\sigma,0}$

For $i = n-1, \ldots, 0$:

1. $x_i \leftarrow \langle \mathbf{t_i}, \mathbf{b_i} \rangle / \|\mathbf{b_i}\|^2$
2. Pick $z_i \sim D_{\mathbb{Z},\sigma',x_i-\lfloor x_i \rceil}$ with $\sigma' := \sigma / \|\mathbf{b_i}\|$, and let $k_i \leftarrow \lfloor x_i \rceil + z_i$
3. Let $\mathbf{t_i} \leftarrow \mathbf{t_{i+1}} - k_i \mathbf{b_i}$ and $\mathbf{v_i} \leftarrow \mathbf{v_{i+1}} + k_i \mathbf{b_i}$

return $\mathbf{v}_0$

- z_i is sampled by `SamplerZ` who calls `BaseSampler`
- Basic GPV: $\mathbf{v} = \lfloor \mathbf{cB}^{-1} \rceil \mathbf{B}$
- `ffSampling`: $\mathbf{v} = (\lfloor \mathbf{cB}^{-1} \rceil + z_i)\mathbf{B}$



Summary of attack on BaseSampler

Algorithm 2: `BaseSampler` ()

Output : An integer $z^+ \sim D_{\mathbb{Z}^+,\sigma_{\max},0}$

1 $u \leftarrow \texttt{UniformBits}(72)$
2 $z^+ \leftarrow 0$
3 **for** $i \leftarrow 0$ **to** 17 **do**
4 $\quad z^+ \leftarrow z^+ + [\![u < \mathrm{RCDT}[i]]\!]$
5 **end**
6 **return** z^+

- $z^+ = 0 \rightsquigarrow z_i \in \{0, 1\} \rightsquigarrow y_i \in (-1, 1] \implies s \in \mathcal{P}(\tilde{B})$ in order to apply HPP solver.

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- $z^+ = 0 \rightsquigarrow z_i \in \{0, 1\}$ since in SamplerZ implementation, $z_i \leftarrow b + (2 \cdot b - 1)z^+$, for $b = 0$ or 1 with equal probability.

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- $z_i \in \{0,1\} \rightsquigarrow y_i \in (-1,1]$ since $\mathbf{s} := (\mathbf{t} - \mathbf{v}) \cdot \mathbf{B}$, $x_i := \langle \mathbf{t}, \tilde{\mathbf{b}}_i \rangle / \|\tilde{\mathbf{b}}_i\|^2$:

$$\mathbf{s} = \sum_{i \in [n]} y_i \cdot \tilde{\mathbf{b}}_i \quad \text{where } y_i = z_i - x_i + \lfloor x_i \rfloor.$$

Summary of attack on BaseSampler

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Output : An integer $z^+ \sim D_{\mathbb{Z}^+,\sigma_{\max},0}$

```
u ← UniformBits(72)
z⁺ ← 0
for i ← 0 to 17 do
    z⁺ ← z⁺ + ⟦u < RCDT[i]⟧
end
return z⁺
```

- $z^+ = 0 \rightsquigarrow z_i \in \{0,1\} \rightsquigarrow y_i \in (-1,1] \implies s \in \mathcal{P}(\tilde{B})$ in order to apply HPP solver.
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$$\mathbf{s} = \sum_{i \in [n]} y_i \cdot \tilde{\mathbf{b}}_i \quad \text{where } y_i = z_i - x_i + \lfloor x_i \rfloor.$$

- but distrib(y_i) is not uniform over $(-1,1] \rightsquigarrow$ cannot directly apply HPP solver.

HPP

Let $\mathbf{B} := (\mathbf{b_0}, ..., \mathbf{b_{n-1}})$ a basis of n linearly independent vectors and let $\mathcal{P}(\mathbf{B}) = \{\sum_{i=0}^{n-1} x_i \mathbf{b_i},\ x_i \in [-1, 1]\}$, the parallelepiped spanned by $\mathbf{B}$. Given a sequence of poly(n) independent samples drawn uniformly at random in $\mathcal{P}(\mathbf{B})$, find a good approximation of $\mathbf{B}$.

Takeaways:

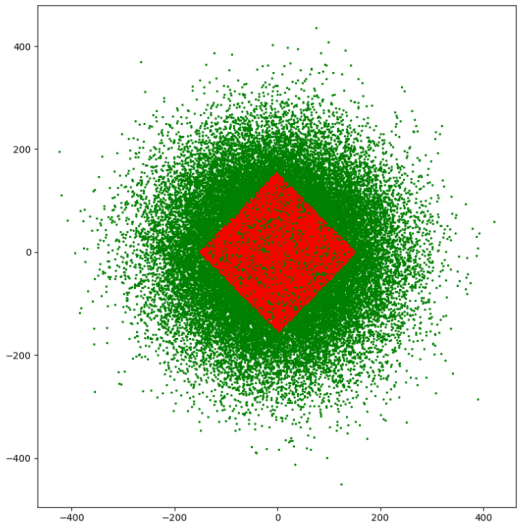
- A HPP solver proposed by Nguyen and Regev in [NR06] breaks NTRUSign and GGH signatures
- Falcon's signature are very similar to these signatures (GPV framework)
- Can we apply directly these solver to Falcon's signature ?

No, because of random perturbation added in their computation ☹

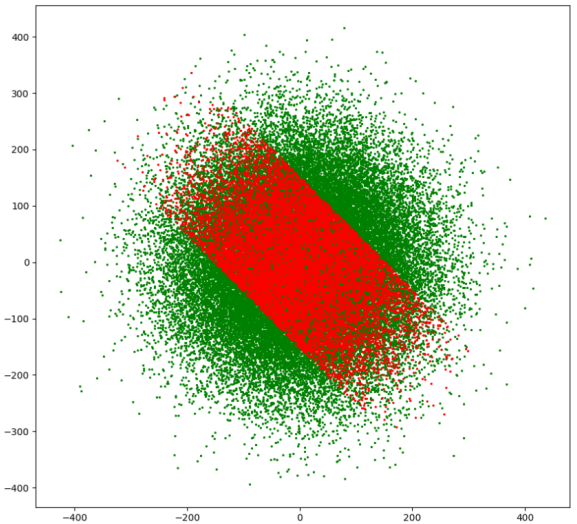
ffSampling & HPP - HPP Deformed variant

- Ducas and Nguyen in [DN12] extended the HPP attack to instance where a small perturbation is added $\delta \in [-1, 1]^n$
- The vector v becomes $(\lfloor \mathbf{cB}^{-1} \rceil + \delta)\mathbf{B}$
- if the perturbation is *partial* (i.e. some set of fixed coordinates indexes have $\delta = 0$), then the HPP solver can recover some $\pm\mathbf{b_i}$
- In our problem, the perturbation δ is z_i

$\Rightarrow$ We can apply the HPP solver on each index i by filtering signatures for which $z_i = 0$



(a) Signatures where $\mathbf{z} \in \{0, 1\}^2$ are in red



(b) Signatures where $z_0 \in \{0, 1\}$ are in red

ffSampling & HPP - Note

- $P[z_i \in \{0,1\}] \approx \text{erf}(\frac{\sqrt{2}}{2\sigma_i}) \in [0.4111, 0.5613]$

$\Rightarrow$ 41% to 56% signatures are kept for each index

- In practice, Falcon's signature are based on the Gram-Schmidt Orthogonalization (GSO) $\tilde{\mathbf{B}}$ rather than $\mathbf{B}$.
- HPP solver will return rows of $\tilde{\mathbf{B}}$
- Falcon use `ffLDL` algorithm for GSO which preserves the first three rows: $\tilde{\mathbf{b_0}}, ..., \tilde{\mathbf{b_3}} \approx \mathbf{b_0}, ..., \mathbf{b_3}$

$\Rightarrow$ $\tilde{\mathbf{b_0}} = \mathbf{b_0} = (g'_0, ..., g'_{n-1}, -f'_0, ..., -f'_{n-1}) = (g', -f')$, an approximation of the private key

Recovering the key - Two options

Direct Recovering (mere rounding):

- Lot of signature measurements: $+5$ mio for a high probability (> 0.99) to have an absolute error less than 0.5 on each coefficients
- Easy to compute

Lattice Reduction (Work/Measurement trade-off):

- Less signature measurements
- Solve lattice reduction problem using Leaky LWE / NTRU tool

$\Rightarrow$ More work and computation time

$\Rightarrow$ 1 mio measurements $\approx$ 1000 hours

$\Rightarrow$ With $+1.5$ mio, it becomes reasonable

Proposed countermeasure

Algorithm 3: Proposed countermeasure to mitigate our attack. This pseudocode corresponds to the last comparison during the computation of $[\![u < \mathrm{RCDT}[i]]\!]$ at line 4 of `BaseSampler`

Input : Two 24-bit variables $\overline{u}$ and $\overline{\mathrm{RCDT}[i]}$ stored in 32 bit registers.
a bit c carrying the result of the comparison of both least significant registers (corresponding to the 48 least significant bits of $u - \mathrm{RCDT}[i]$).
Output : 1 if $\overline{\mathrm{RCDT}[i]} + c > \overline{u}$ and 0 is $\overline{u} \leq \overline{\mathrm{RCDT}[i]} + c$

1 $b \leftarrow$ `0xffffff`
2 $b := b - \overline{u} + \overline{\mathrm{RCDT}[i]} + c$
3 **return** $b \gg 24$

- Replace *underflow* by *overflow*
- if $\overline{RCDT[i]} + c > \bar{u}$, it overflows and returns 1 (0 otherwise)

\+ Reduces the Hamming weight by a factor of 8

\+ Simple sufficient mitigation for this attack

– Weak security assurance (provable masked implementations would be better)

Recent work on Side Channel Attacks against Falcon

Thorough Power Analysis on Falcon Gaussian Samplers and Practical Countermeasure

Xiuhan Lin[1], Shiduo Zhang[2], Yang Yu[2,3,4(✉)], Weijia Wang[1,4], Qidi You[5,6], Ximing Xu[7], and Xiaoyun Wang[1,2,3,4]

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[Lin et al.]

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Masking Floating-Point Number Multiplication and Addition of Falcon

First- and Higher-order Implementations and Evaluations

Keng-Yu Chen[1] and Jiun-Peng Chen[1,2]

[1] National Taiwan University, Taipei, Taiwan, r11921066@ntu.edu.tw
[2] Academia Sinica, Taipei, Taiwan, jpchen@ieee.org

Keng-Yu Chen et al. [CC24]

Summary[1/3]

In this talk we have seen...

- **Side-channel security of Falcon**
  - floating-point operations = targets for SCA $\rightsquigarrow$ pre-image attack [KA21], improved by [GMRR22] (reduced number of traces and complexity)
  - GMRR22 provided the 1rst Power analysis using the leakage in BaseSampler and HPP solver.

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In this talk we have seen...

- **Side-channel security of Falcon**
  - floating-point operations = targets for SCA $\rightsquigarrow$ pre-image attack [KA21], improved by [GMRR22] (reduced number of traces and complexity)
  - GMRR22 provided the 1rst Power analysis using the leakage in BaseSampler and HPP solver.
- **Two attacks by [GMRR22]:**
  - Partial knowledge of $\hat{f}$ leads to KR.
  - power consumption leakage $\rightsquigarrow$ noting when z^+ is incremented $\rightsquigarrow$ noting when $z_i \in \{0, 1\} \rightsquigarrow$ collecting some $y_i \in (-1, 1] \implies s \in \mathcal{P}(\tilde{B}) \implies$ apply HPP or Deformed HPP solver $\implies$ KR.

Summary [2/3]

- 1rst Attack (on preimage computation): interesting for anyone looking to perform SCA on polynomial multiplication on floating points.
- 2nd Attack (on BaseSampler) worked because: $u < \text{RCDT}[i]$ is a 72 bits substractions $\rightarrow$ hard to perform on a device with only 32-bit registers. But Falcon allows to do three 24 bits substractions $\rightsquigarrow$ SCA works: noting when z^+ is incremented.

Summary[3/3]

-  These new attacks highlight the need for Side-channel protection for one of the 3 finalists of NIST's standardization campaign.
- **Recent works:** [CC24] provided the first masking floating-point multiplication and addition which protects Falcon's pre-image computation against the attack of [KA21]
- **Countermeasures:** effective and easy-to-implement countermeasures against leakages to protect Falcon's integer Gaussian sampler (Lin et al.)

Thank you!

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